Ole Rømer's temperature correction of pendulum clock rates

Poul Darnell*

The Danish astronomer Ole Rømer (1644–1710) had a preferred observation method which called for the use of pendulum clocks. This article describes how Rømer tried to improve the accuracy of his observations by correcting the performance of his clocks. Rømer realized that these clocks ticked with a slightly different rhythm during the day and the night due to variations in ambient temperature. His scientific diary, the Adversaria, shows that he sought to compensate for this source of error, as he experimented with the effect of temperature variation on the lengthwise dilation of various metals. This in turn led him to experiment with thermometers, and to suggest a thermometric scale with two fixed points.

On 21 April 1703 Ole Rømer wrote a letter to his friend, the German mathematician and philosopher Gottfried Wilhelm Leibnitz.[1] In this letter Rømer described his transit telescope (Fig. 1) which he had installed in his private home.[2] The letter also included a description of the pendulum clocks, which were used together with the transit instrument. Rømer wrote:

> The instrument has three three-foot pendulum clocks, which I know so well after twenty years of use, so that in 24 hours' time, they only cheat the observer half a second. It would lead too far to give a detailed account of the efforts I have made to avoid, correct or at least detect the many small errors, which tend to appear in all forms of mechanics because of defects in materials and in operations and the constant change of weather conditions.

This interesting statement tells us that Rømer fully believed that he was able to control his timekeeping within half a second per day, which at that time was an amazingly good result - but he does not explain how this was done.

Rømer's preferred observation method was to determine a star's right ascension and declination.[3] The declinations were determined by his transit telescope (later by his meridian circle), the right ascensions by his clocks only. For this reason his observation accuracy was directly dependent on the performance of his clocks.

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1. *Ole Rømer, Korrespondance og afhandlinger samt et udvalg af dokumenter*, Per Friedrichsen and Christian Gorm Tortzen, eds. (Copenhagen: Det Danske Sprog- og Litteraturselskab, C.A. Reitzels Forlag, 2001), p. 307, letter no. 58.

2. The engraving dates from 1704. All four engravings reproduced in this article were commissioned by Rømer who intended to publish a book on his astronomical work and instruments, but he never managed to do so. The engravings were published for the first time by Peder Horrebow in his *Basis Astronomiæ, sive Astronomiæ Pars Mechanica* (Copenhagen, 1735).

3. Letter to G.W. Leibnitz dated 24 January 1700, in *Ole Rømer, Korrespondance*, p. 268, letter no. 46. The co-ordinate system used by astronomers has axes known as right ascension (RA) and declination (Dec) (A. E. Roy, D. Clarke, *Astronomy – Principles and Practice*, University of Glasgow, 1977). These can be compared to the co-ordinate system of longitude and lattitude for expressing a particular position on the Earth's surface. Rømer's observatories, equipped with clocks, transit telescope and meridian circle, were specially designed for observations of RA and Dec.

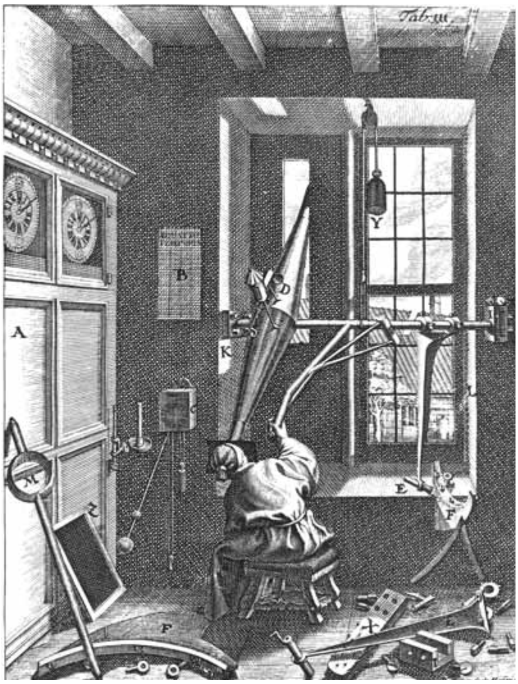
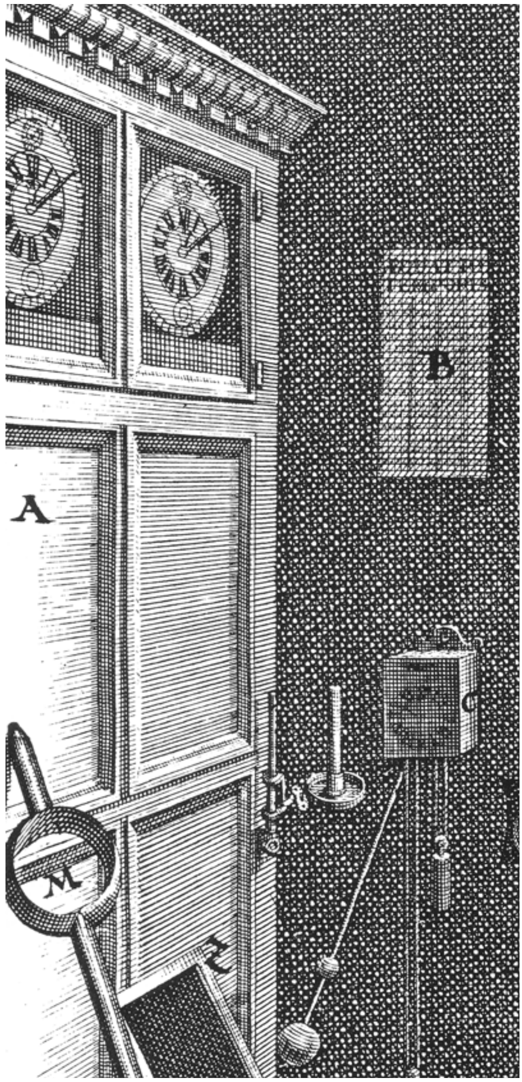


Fig. 1. Rømer at the transit instrument in his home, with the three clocks shown in detail. Horrebow, *Basis Astronomiae* (1735), plate III.



In this connection it is clear that Rømer in his transit telescope also had the perfect tool for control of his clocks, as the time between two successive passages in his transit telescope of a certain fixed star is exactly 24 hours (sidereal time).

Rømer had noticed that his clocks were gaining during nighttime and losing during daytime. He explained this by variations in the pendulum lengths.[4] In order to improve his observation accuracy, he therefore had to do the following:

- Measure the temperature variations in his observatory.
- Measure the thermal expansion of his pendulums and find out how the pendulum lengths varied with the temperature variations.

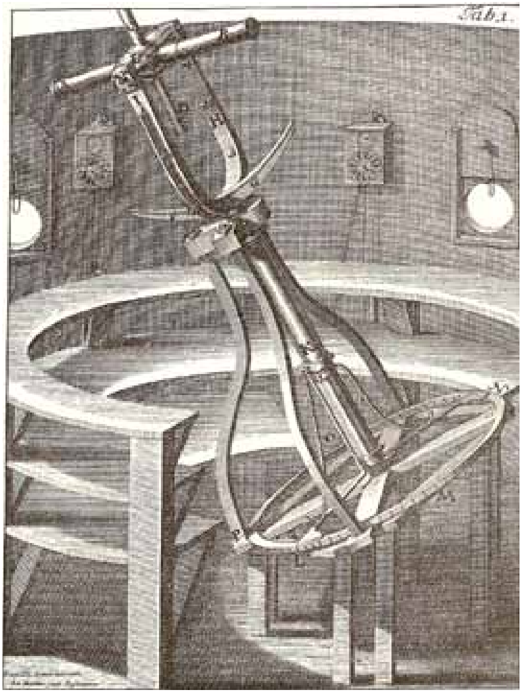
If he would know exactly how the temperature changes influenced his clocks rates, he could afterwards correct his observations. In the following I will try to explain how Rømer tried to fulfill this.

Observation clocks

The information about Rømer's clocks is taken from two sources. The first is a book, *Basis Astronomiæ, sive Astronomiæ Pars Mechanica,* written by his pupil and successor as professor of astronomy, Peder Horrebow and published in 1735 in Copenhagen. It contains a description of Rømer's methods, instruments and projects. In chapter 3, Horrebow gives a comprehensive description of the observation clocks. The second is a scientific-technical scrapbook named *Adversaria*,[5] one of the very few

4. Horrebow, *Basis Astronomiæ*, §162.

5. *Adversaria*, Thyra Eibe and Kirstine Meyer, eds. (Copenhagen: Det Kgl. Danske Videnskabernes Selskab, 1910).



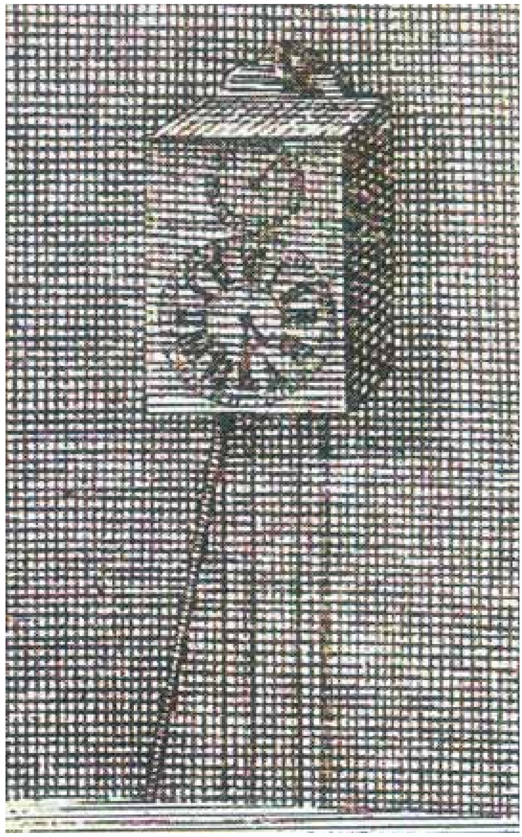


Fig. 2. Rømer's equatorial instrument, with one of the two clocks shown in detail. Horrebow, *Basis Astronomiae* (1735), plate I.

documents Rømer left behind, as nearly all his instruments and observations journals were destroyed in the great fire of Copenhagen in 1728.

Horrebow describes the clocks as:

> Huygens', where the three foot pendulum for each swing indicates one second, and which gives a swing of six, seven or eight inches from the vertical position.[6]

Christiaan Huygens is mentioned because the clocks were of his design, and because Huygens and Rømer had been working closely together during Rømer's stay at the Académie des Sciences in Paris in the years 1672 to 1681.

In *Adversaria,* Rømer writes that the clock pendulums have a length of about 38 Danish inches, or 456 lines, which is exactly the length of the one-second pendulum of 994 mm.[7]

Three astronomical observatories

The three astronomical observatories that Rømer had at his disposal reflected the development of his astronomical instruments. The first was the Copenhagen University Observatory on top of the Round Tower in Copenhagen. The second was in his private home only a few hundred meters west of the Round Tower. The third was established about two miles west of Copenhagen near the village Vridsløsemagle, and it was here that the world's first meridian circle was installed in 1704. For practical reasons Rømer had placed the three observatories on the same latitude, which made it easier to compare the observations results from the three sites.

Figs 2 and 3 show the equatorial instrument and the azimuthal instrument, both installed at the Round Tower by Rømer after he became professor of astronomy in

6. Horrebow, *Basis Astronomiæ*, §30.

7. *Adversaria*, p.121. The Danish inch, introduced by Rømer in the period 1681-98, was 26.1725 mm.

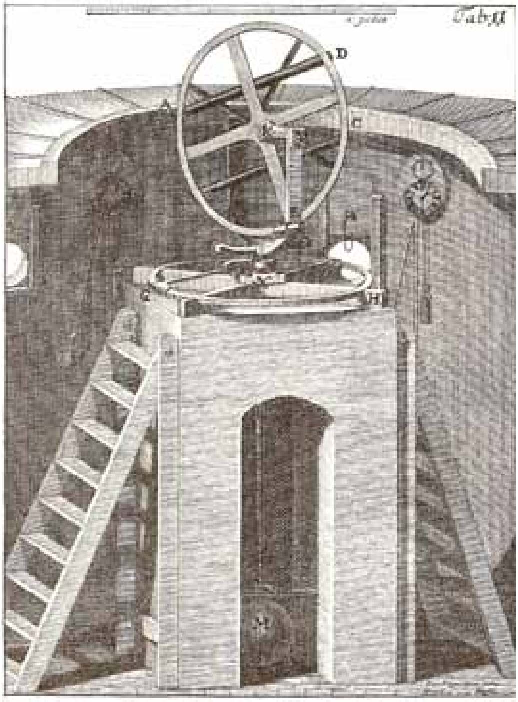
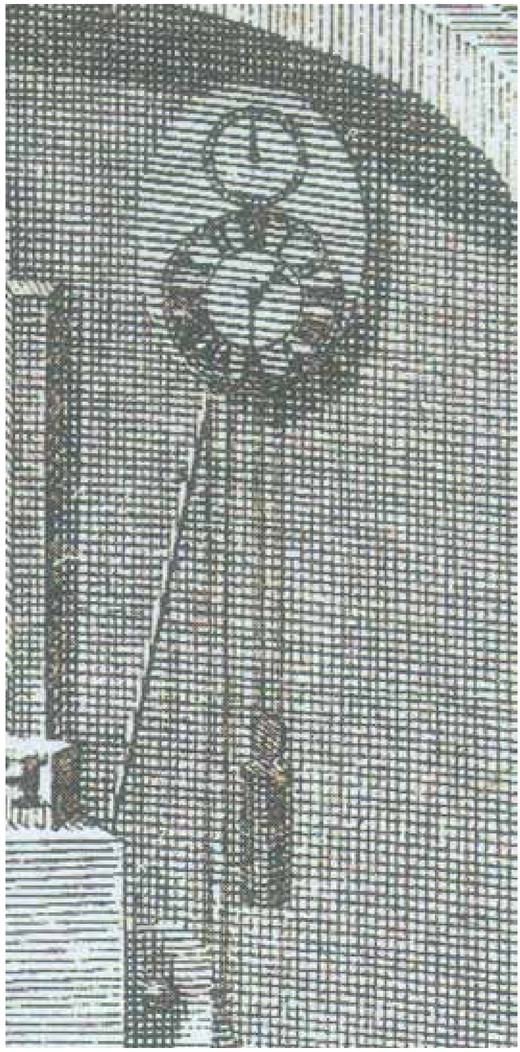


Fig. 3. Rømer's azimuthal instrument, with one of the two clocks shown in detail. Horrebow, *Basis Astronomiae* (1735), plate II.

Almanak
for det Aar efter
Christi Fødsel 1782.
som er det andet Aar efter Skud-Aar.
Beregnet og fastenstevet
til Kiøben- hauns Poli-
Høyde 55 Grader
40 Minuter. 56 Sekunder.
Af
Thomas Bugge.
Mathem. & Astr. Prof. Publ. Ord.
Kiøbenhavn,
trykt hos Directeur N. C. Høpffner, Hans Kongl.
Majestæts og Universitets Bogtrykker,
og findes hos hannem tilkiøbs.

Fig. 4. Front page of the Danish almanac for 1782, showing the Round Tower with the two domes housing Rømer's equatorial and azimuthal instruments. Rømer's observatory was shown on the front page of the almanac throughout the years 1701–1855, although the buildings were changed during that period.



1686. Each was housed in a separate dome, and each had two separate clocks with different shape of the dials. The engravings show that the top dial had a second hand, and the bottom dial minute and hour hands. The Round Tower, with Rømers' observatory on its top, was illustrated on the front page of the Danish almanac (Fig. 4).

The next instrument that Rømer installed was the transit telescope in his private home in 1690. This instrument (Fig. 1) was provided with three pendulum clocks. The clock to the left of the telescope is the one which the observer is looking at, while at the same time looking into the telescope and listening to the pendulum beats. The two clocks in the cabinets were called 'The Sun Clock' and 'The Moon Clock', and they

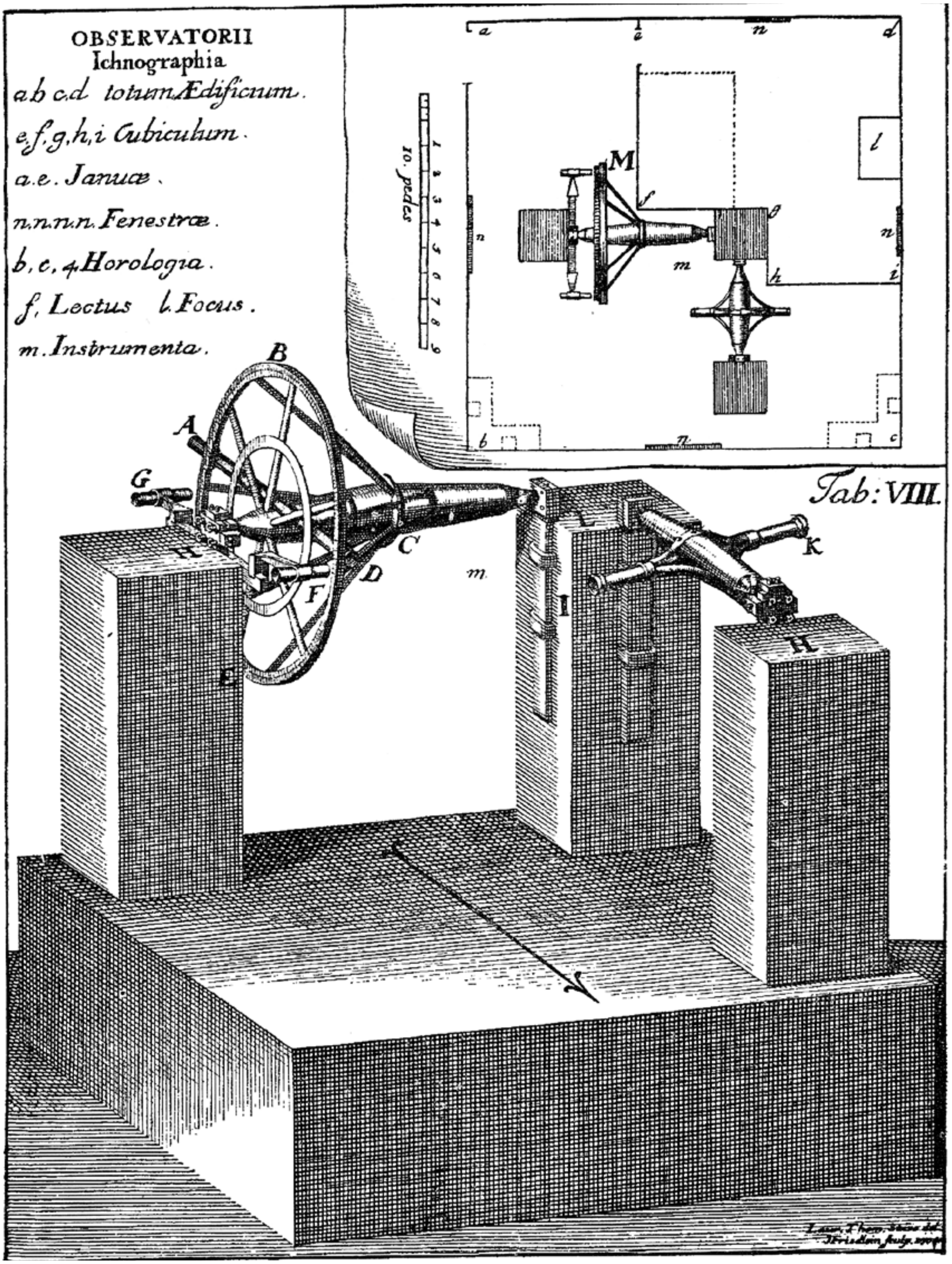


Fig. 5. Rømer's meridian instrument. Horrebow, *Basis Astronomiae* (1735), plate VIII.

were only used for control of the first clock, as their performance was also noted by the observer. The clocks were placed in separate cabinets in order to reduce influence from the surroundings.

In 1704 Rømer installed his meridian circle (Fig. 5) at Vridsløsemagle. This was a more precise instrument than his transit telescope, but was also used for determination of star positions. A transit instrument was installed at right angles to the meridian circle, but this instrument was used for equinox observations only. The two instruments were placed in a small hut; to allow the observations to be made slits had been cut through the walls and the roof. Each instrument had two clocks, as indicated on the engraving (b & c). All we know about these clocks is that one is designated as 'the light one' and the other as the 'heavy one'. According to Horrebow this refers to the weight of the clocks.[8]

Because of the long distance between the Round Tower in Copenhagen and the observatory at Vridsløsemagle, Rømer used hollow mirrors to send light signals for synchronizing the clocks. This is known from the inventory list of the Round Tower drawn up in 1710, and from a report by a Swedish spy who visited the tower in 1708. In that report he explained how a steel mirror (diam. 36 inch) with a lamp in front was used during nighttime to send light signals at pre-arranged times for the correction of the meridian circle clocks.[9] It is known that Rømer had several students who assisted him in his observation work, and it is likely that some of these had the task to send the signals.

Measuring the temperature

Rømer has described his alcohol filled thermometers in *Adversaria* in 1702.[10]

7½° Rø was defined as the freezing point for water and the boiling point of water was 60° Rø. As the difference between the two fixed points is 52½° Rø, this means that 1° Rø = 1.905° C.

An eyewitness tells us about Rømer's manufacture of thermometers. In a letter dated 17 April 1729, Daniel Gabriel Fahrenheit wrote to Professor Herman Boerhaave in Leiden:

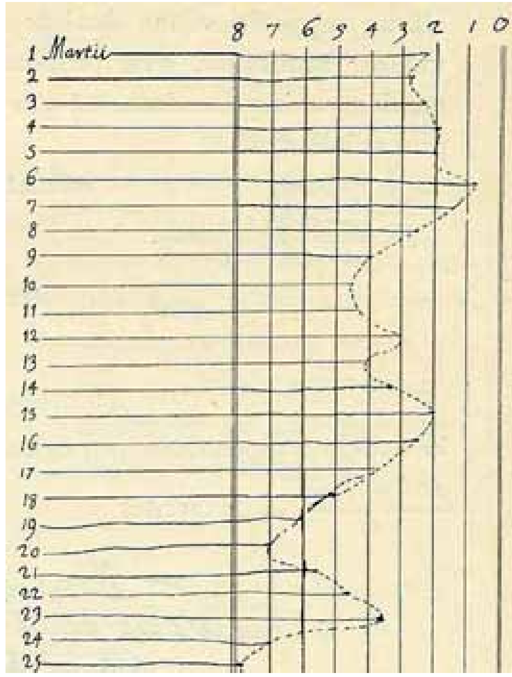


Fig. 6. Part of a curve showing Rømer's recording of the temperature variations in Copenhagen for the winter 1708–1709. From *Adversaria*.

> As to the means whereby I came to begin improving thermometers, it may be useful for you to know that it was the commerce I had with the excellent Mr. Rømer of Copenhagen in the year 1708 that first led me in this direction, for arriving at his house one morning I found that he had several thermometers standing in water and ice, which he later placed in warm water […].'[11]

The temperature curve from *Adversaria* (Fig. 6) shows that Rømer did use his thermometers to register temperature variations.[12]

8. Horrebow, *Basis Astronomiæ*, §371.

9. *Dansk Astronomi Gennem Firehundrede År*, Claus Thykier, ed. (Copenhagen: Rhodos, 1990), Vol.3, p. 435. At that time Sweden was Denmark's enemy, and the Round Tower was a good place for a spy who wanted to count the Danish warships in the harbour.

10. *Adversaria*, pp. 202-14.

11. *Fahrenheit's Letters to Leibnitz and Boerhaave*, Pieter van der Star, ed. (Amsterdam:Rodopi, 1983), p. 171.

12. *Adversaria*, p. 214.

Thermal expansion of pendulums

We know from *Adversaria* that in 1692 Rømer measured the coefficients of thermal expansion for different materials.[13] To be certain of the results, Rømer repeated the measurements three or four times. The rods used had a length of three feet and a square cross section of ¼ inch. Each rod was divided into 6800 equal parts. The rods were heated from 6½^{O} Rø to 30½^{O} Rø, and the following expansions were found for these materials:[14]

Gold and copper	5 parts
Silver and pewter	6½ parts
Lead	9½ parts
Iron	3½ parts
A round glass rod	3½ parts

Temperature heating difference: 30.5 – 6.5 = 24^{O} Rø. As 100^{O} C = 52½^{O} Rø, the coefficients of thermal expansion can be calculated. For example for iron: 3.5 x 52.5 : 6800 x 24 x 100 = 1.1 x 10^{-5} = 0.000011. The modern value for iron is 0.000012. Regrettably Rømer did not describe which equipment he used to make the measurements, so we have to assume that the deviations from the modern values are due to the fact that the test materials contained too many impurities.

And then at last, after having made thermometers and having determined the coefficient of expansion for his pendulum rod of iron, Rømer was able to calculate the result of the expansion of his 1-second pendulums. The result was that his clocks would gain or lose 1 second in 24 hours for a change of 1^{O} Rø (see appendix for details). This calculation is stated in *Adversaria* in the text, where he describes his investigation of the coefficients of thermal expansion.

Now finally Rømer could start to correct his observations results by using a temperature curve showing the variations in his observatory over time.

13. *Adversaria*, pp. 119-21.

14. Mogens Pihl, *Ole Rømers videnskabelige liv* (Copenhagen, 1944), pp. 49-50.

Appendix. Verification of Rømer's result: his clocks would gain or lose 1 second in 24 hours' time per 1^{O} Rø

Coefficient of thermal expansion:	α
Original length:	l_0
Expansion:	Δl
Temperature difference:	t
Changed rate in 24 hours in seconds:	ΔT

<u>Formulas used</u>:

Expansion: $\Delta l = l_0 \times \alpha \times t$

New length: $l_1 = l_0 \pm \Delta l$

(A shorter pendulum will gain and a longer lose in time)

Changed rate in 24 hours in seconds:
$\Delta T = (\Delta l \times 86400)/(2 \times l_0)$

24h = 60 x 60 x 24 = 86400s

<u>Option for Rømer's value of the coefficient</u>:

Coefficient of thermal expansion: α = 0.000011 Rømer's value

Original length: l_0 = 994 mm

Temperature difference: t = 1.905^{O} C equal to 1^{O} Rø

$\Delta l = l_0 \times \alpha \times t = 994 \times 0.000011 \times 1.905 = 0.0208$ mm

$\Delta T = (0.0208 \times 86400)/(2 \times 994) =$ **0.9053≈ 1 s pr. 24 h.**

<u>Option for modern value of the coefficient</u>:

Coefficient of thermal expansion: α = 0.000012 modern value

Original length: l_0 = 994 mm

Temperature difference: t = 1.905^{O} C equal to 1^{O} Rø

$\Delta l = l_0 \times \alpha \times t = 994 \times 0.000012 \times 1.905 = 0.0227$ mm

$\Delta T = (0.0227 \times 86400)/(2 \times 994) =$ **0.9875 ≈ 1 s pr. 24 h.**